AP Calculus AB
Unit 5 Review

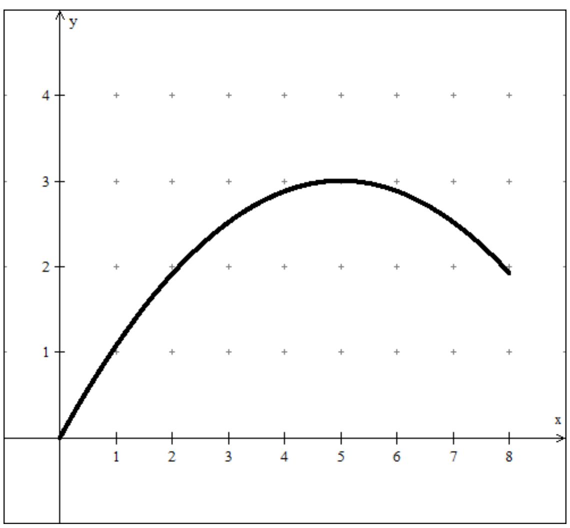
Name: ____________________________
Date: ________________ Block: _____

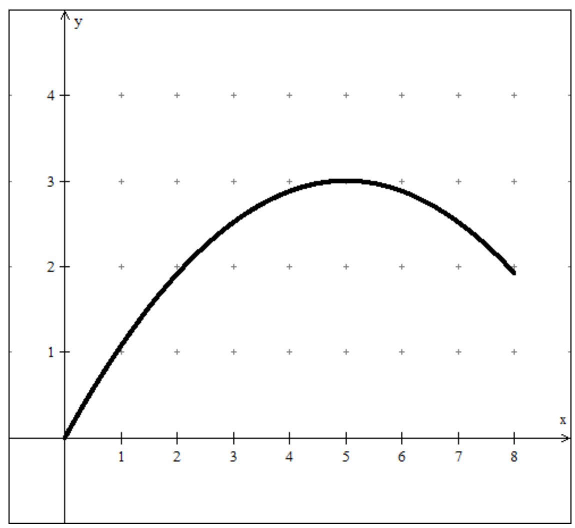
1.) Find $\frac{d}{dx}\int_x^1 (1-t^3)dt$

2.) $\frac{d}{dx}\int_2^{x^2} (1-t^3)dt$

3.) Find the interval on which $f(x) = \int_0^x (t^3+t^2+1)dt$ is concave up.

4.) Evaluate using Riemann Sums L_4, R_4 for the following on the interval $0 \le x \le 8$ with four subintervals.





5.) Approximate $\int_2^4 (3x^2-2x)dx$ using

a.) Midpoint rule with 4 subintervals

b.) Trapezoid rule with 4 subintervals

6.) The function $f(x)$ is increasing and concave down on the interval $[0, 6]$ using three subintervals of equal length:

a.) Is the approximation L_3 an overestimate or underestimate of $\int_0^6 f(x)dx$? Justify your answer.

b.) Is the approximation R_3 an overestimate or underestimate of $\int_0^6 f(x)dx$? Justify your answer.

7.) A particle's velocity, given in feet per second, is described by the equation $v(t) = 3t^2 - 12t + 9$ when $0 \le t \le 4$. Find:

a.) The total distance traveled on the given time interval.

b.) The particle's displacement after 4 seconds.

8.) Given $a(t) = 12t^2 - 2$, $v(2) = 25$, and $s(2) = 4$, write an expression for $s(t)$.

9.) Given $\int_0^3 f(x)dx = 7$ and $\int_0^5 f(x)dx = 13$

a.) $\int_3^5 (f(x) + 2)dx$

b.) $\int_0^3 (4f(x) + x)dx$

c.) $\int_5^{10} f(x - 5)dx$

d.) $\int_3^3 f(x)dx$

Evaluate each of the following integrals #'s (10-21).

10.) $\int x\sqrt{x+2}dx$

11.) $\int \sin(2x)\sqrt{\cos(2x)-3}dx$

12.) $\int\left(x^2-3x+\frac{1}{x}+\frac{2}{x^2}\right)dx$

13.) $\int \frac{x^4+x^2-3}{x^2}dx$

14.) $\int_3^{11}\frac{dx}{\sqrt{2x+3}}$

15.) $\int \frac{\cos(\ln x)}{x}dx$

16.) $\int_0^3 \frac{x^3}{x^4+1}dx$

17.) $\int_2^5 (x^3-2x^2+3x^{-1})dx$

18.) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$

19.) $\int \frac{x+1}{x^2+1}dx$

20.) $\int\left(\sqrt{t}-\frac{1}{\sqrt{t}}\right)^2 dt$

21.) $\int e^{3\ln u}du$

22.) If $y=f(x)$ is a solution to the differential equation $\frac{dy}{dx}=xe^{x^2}$ with the initial condition $f(0)=2$, find the particular solution $f(x)$.

For 23-24

a.) Find the average value of f on the give interval.
b.) Calculate the value of c that satisfies the MVT for Integrals.

24.) $f(x)=2+3x$ on $[0,4]$

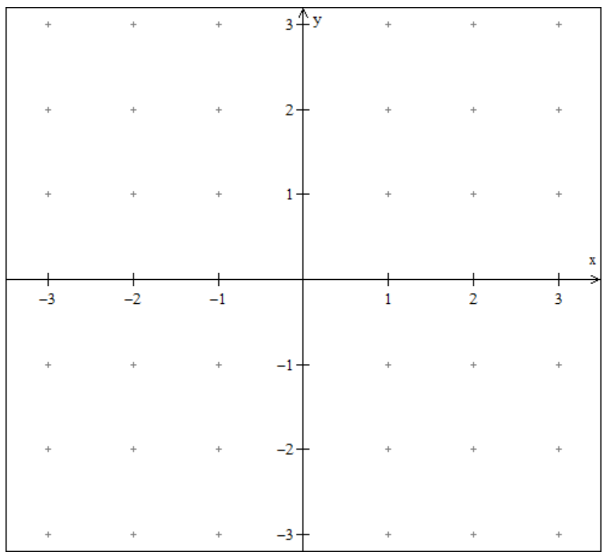
25.) $f(x)=\sin(3x)$ on $[0,\pi]$

26.)

a.) If x represents the number of units produced for a certain commodity and $C'(x)$ is the rate of change of cost with respect to x, what are the units of $C'(x)$?

b.) What does $\int_a^b C'(x)dx$ represent?

27.) a.) Draw a slope field for $\frac{dy}{dx} = \frac{y}{x}$



b.) Solve for the solution curve that passes through $(1, 2)$

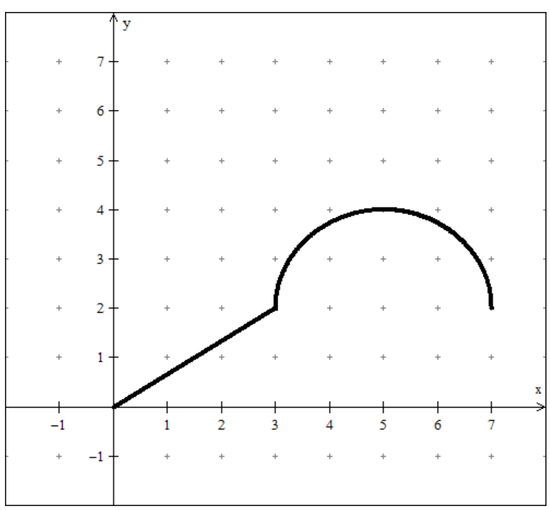
t (minutes)	0	4	7	9
$r(t)$ (gallons per minute)	9	6	4	3

28.) Water is flowing into a tank at the rate $r(t)$, where $r(t)$ is measured in gallons per minute and t is measured in minutes, the tank contains 15 gallons of water at time $t = 0$. Value of $r(t)$ for selected values of t are given in the table above. Using a trapezoidal sum with 3 intervals, indicated by the table what is the approximation of the number of gallons of water in the tank at time $t = 9$?

29.) Let h be the function defined by $h(x) = \int_{\frac{\pi}{4}}^{x} \cos^2(t)\,dt$. What is the equation of the tangent to the graph of h at the point where $x = \frac{\pi}{4}$.

30.) If $y = f(x)$ is a solution to the differential equation $\frac{dy}{dx} = \frac{2xy}{x^2+1}$ with the initial condition $f(0) = 1$, find the particular solution $f(x)$.

31.) Given the graph of g below consisting of one line segment and a semi-circle from $[0, 7]$ if $f(x) = \int_3^x g(t)dt$ determine $\lim_{x \to 3} \frac{f(x)}{x^2-9}$.

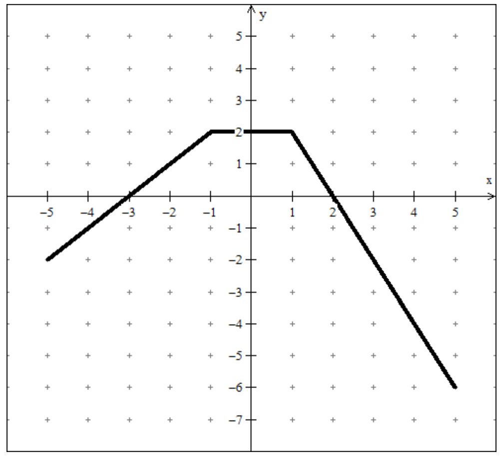


32.) The number of people who have entered a museum on a certain day is modeled by a function $f(t)$, where t is measured in hours since the museum opened that day. The number of people who have left the museum since it opened that same day is modeled by a function $g(t)$. If $f'(t) = 380(1.02^t)$ and $g'(t) = 240 + 240\sin\left(\frac{\pi(t-4)}{12}\right)$, at what time t, for $1 \le t \le 11$, is the number of people in the museum at a maximum?

33.) The functions f and g are differentiable for all real numbers and g is strictly decreasing. The table below gives values of the functions and their first derivatives at selected values of x. Let h be the function given by $h(x) = \int_{-1}^{g(x)} f(t)dt$. Find the value of $h'(-1)$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-2	3	5	4	-3
-1	5	1	0	-2
0	8	-3	-2	-1
1	-2	-4	-3	-2

34.)



Let $f(x)$ be defined by the graph above whose domain is $[-5, 5]$. Let $F(x) = \int_{-3}^{x} f(t)dt$ and $F(-3) = 0$.

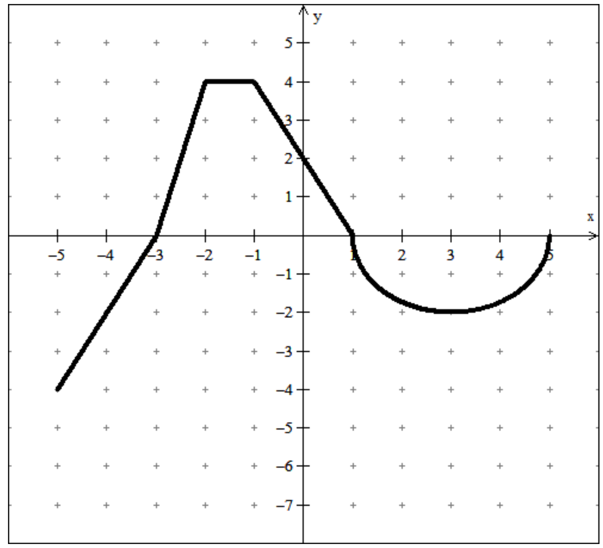
a.) Put $F(4), F'(4)$ and $F''(4)$ in order from least to greatest.

b.) Find the equation of the tangent line to F at $x = 4$.

c.) Use the results of b.) to approximate the value of F at $x = 4.1$. Does this value overestimate or underestimate $F(4.1)$? Justify your answer.

d.) Find the x value(s) where F has a maximum. Justify your answer.

35.) Given the graph of g below consisting of four line segments and a semi-circle on $[-5,5]$, if $f(x)$ is defined as $f(x)=\int_{-1}^{x} g(t)dt$ determine the following.



$f(-3)$ $\qquad$ $f(5)$ $\qquad$ $f(-1)$

$f'(-2)$ $\qquad$ $f'(1)$ $\qquad$ $f'(-5)$

$f''(-4)$ $\qquad$ $f''(3)$

On what intervals is $f(x)$ increasing?

On what intervals is $f(x)$ decreasing?

On what intervals is $f(x)$ concave up?

On what intervals is $f(x)$ concave down?

At what ordered pairs does $f(x)$ have a relative maximum?

At what ordered pairs does $f(x)$ have a relative minimum?

At what ordered pairs does $f(x)$ have an absolute maximum?

At what ordered pairs does $f(x)$ have an absolute minimum?

At what ordered pairs does $f(x)$ have a point of inflection?

Evaluate $\lim\limits_{x\to -1} \frac{x^2-1}{f(x)}$